

Relativistic Quark-Model Results for Baryon Ground and Resonant States¹

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Abstract. Latest results from a study of baryon ground and resonant states within relativistic constituent quark models are reported. After recalling some typical spectral properties, the description of ground states, especially with regard to the nucleon and hyperon electromagnetic structures, is addressed. In the following, recent covariant predictions for pion, eta, and kaon partial decay widths of light and strange baryon resonances below 2 GeV are summarized. These results exhibit a characteristic pattern that is distinct from nonrelativistic or relativized decay studies performed so far. Together with a detailed analysis of the spin, flavor, and spatial structures of the wave functions, it supports a new and extended classification scheme of baryon ground and resonant states into SU(3) flavor multiplets.

Keywords: Relativistic constituent quark model; Baryon properties; Baryon Resonance Decays; Flavor multiplets

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RELATIVISTIC CONSTITUENT-QUARK MODELS

For the results discussed here we employ two kinds of relativistic constituent-quark models (RCQMs). These are the Goldstone-boson-exchange (GBE) RCQM [1] and a variant of the one-gluon-exchange (OGE) RCQM presented in ref. [2]. They are defined through a relativistically invariant mass operator \hat{M} that is treated in the framework of Poincaré-invariant quantum mechanics. The solution of its eigenvalue equation

$$\hat{M}|V, M, J, \Sigma\rangle = M|V, M, J, \Sigma\rangle \quad (1)$$

leads to the mass M and eigenstate $|V, M, J, \Sigma\rangle$ of a baryon ground or resonant state, characterized by intrinsic spin J with z -component Σ ; the $|V, M, J, \Sigma\rangle$ are simultaneously eigenstates of the velocity operator \hat{V}^μ and also of the momentum operator $\hat{P}^\mu = \hat{M}\hat{V}^\mu$.

For our calculations of baryon reactions, in particular of the elastic electromagnetic and axial form factors of the nucleons and the mesonic decays of the baryon resonances considered below, we adhere to the point form of Poincaré-invariant quantum mechanics, since it allows to produce manifestly covariant results. This is essentially a consequence of the generators of Lorentz transformations to be independent of interactions; the only

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interaction-dependent generators of the Poincaré group are the four components of the momentum operator \hat{P}^μ .

The mass-operator eigenvalue equation (1) is solved with the stochastic variational method (SVM) [3], which allows to achieve very accurate mass eigenvalues and also the most general dependence of baryon wave functions on color, flavor, spin, and spatial variables. More details on the formalism can be found, e.g., in ref. [4] and references therein.

BARYON SPECTRA

The invariant mass spectra of the two types of RCQMs are shown in figs. 1 and 2 for the baryon states below ≈ 2 GeV considered here². Some striking features of the spectra are immediately evident. Here, we only hint to the wrong level ordering of the OGE RCQM with regard to the $\frac{1}{2}^+$ Roper resonance $N(1440)$ and the $\frac{1}{2}^-$ resonance $N(1535)$ in the N spectrum (fig. 1) and the failure of both RCQMs in describing the first excitation $\Lambda(1405)$ in the Λ spectrum (fig. 2).

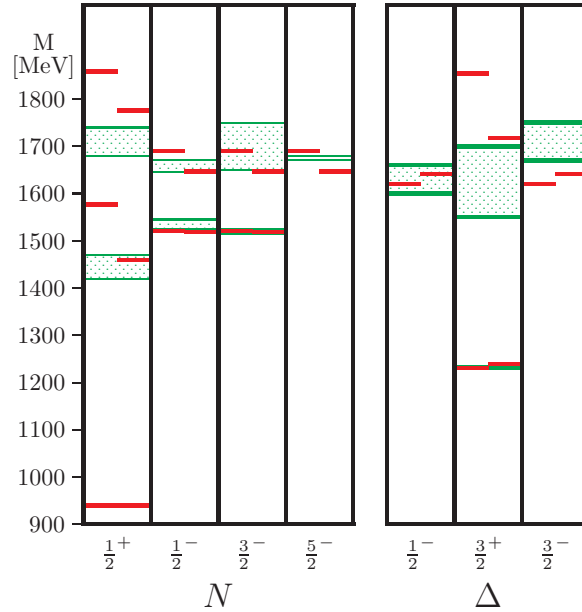


FIGURE 1. Energy levels (red solid lines) of the lowest N and Δ states with total angular momentum and parity J^P for the OGE (left levels) and GBE (right levels) RCQMs in comparison to experimental values with uncertainties [6], represented as (green) shadowed boxes.

² The complete spectra of the RCQMs can be found in refs. [1, 2, 5].

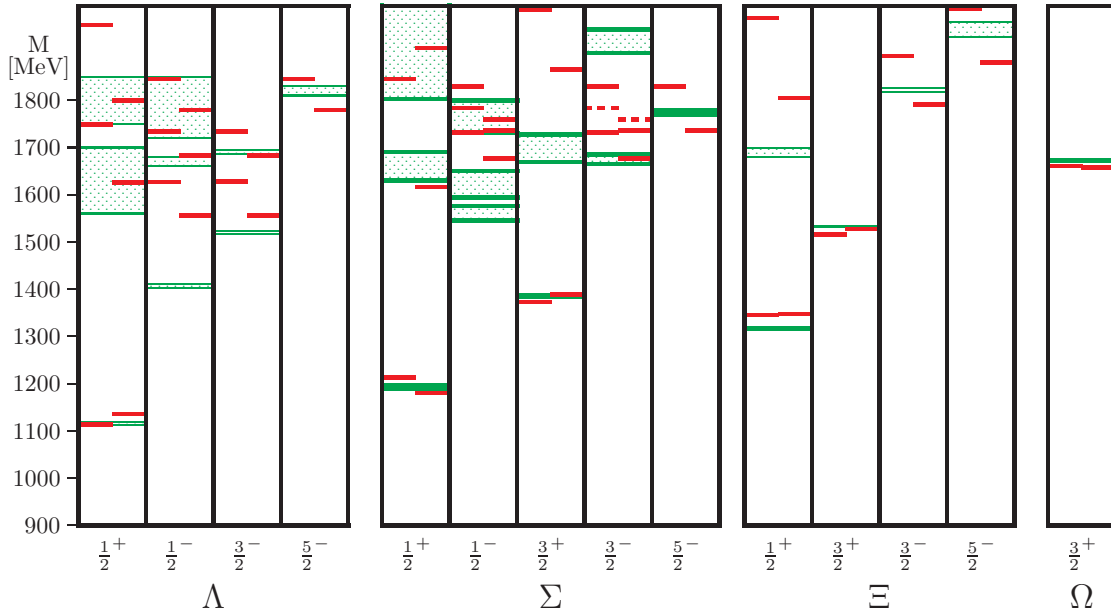


FIGURE 2. Same as in fig. 1 for the lowest Λ , Σ , Ξ , and Ω states. The dashed lines in the $J^P = \frac{3}{2}^-$ Σ spectrum represent (decuplet) eigenstates, for which there is no experimental counterpart yet.

ELECTROWEAK STRUCTURE OF BARYON GROUND STATES

Electromagnetic nucleon form factors

A first test of the mass-operator eigenstates $|V, M, J, \Sigma\rangle$ concerns the elastic electromagnetic form factors of the nucleon. In this context a simplified current operator according to the point-form spectator model (PFSM) has been used so far. It means that the virtual photon couples only to a single constituent quark in the nucleon. Nevertheless the PFSM current represents an effective many-body operator [7].

The covariant predictions of the GBE and OGE RCQMs for the electric and magnetic form factors of the proton and the neutron are shown in figs. 3 and 4. For momentum transfers up to $\approx 4 \text{ GeV}^2$, where we may assume a quark-model description to be reasonable, the results of both the GBE and OGE RCQMs are found in surprisingly good agreement with the available experimental data. A nonrelativistic calculation along the usual impulse approximation fails completely [8, 9]. Similarly a relativistic calculation performed along an instant-form spectator model, constructed in analogy to the PFSM, is found to be far off the experimental data [7]. While the detailed behaviour of the nucleon wave function is of lesser influence, the correct treatment of relativistic effects appears to be most essential. This observation is further supported by the comparison with the relativistic results for the nucleon form factors obtained by the Bonn group with their instanton-induced (II) RCQM [10] in a completely different Bethe-Salpeter approach [11] (also shown in figs. 3 and 4). On the other hand, a simplistic nucleon wave function, such as the one obtained with a confinement potential only, is also not adequate. Since it misses important mixed-symmetric spatial components, it yields an

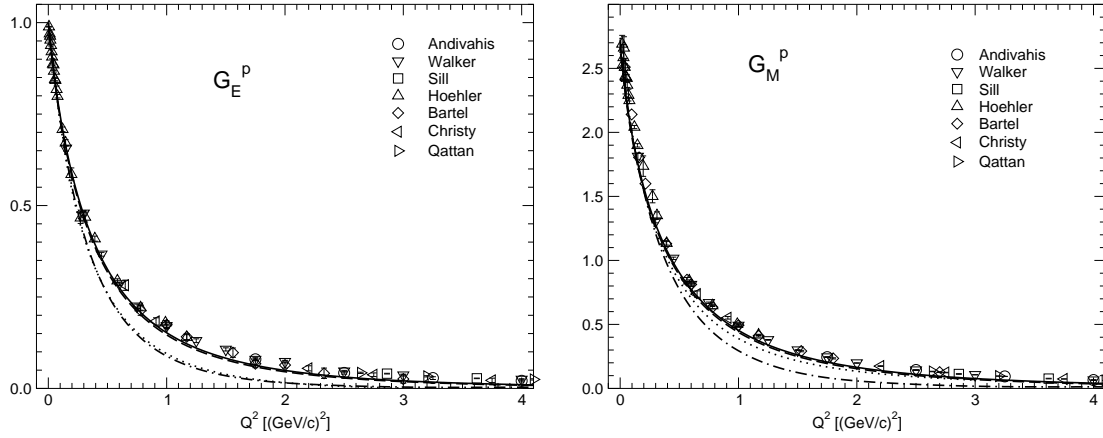


FIGURE 3. Electric and magnetic form factors of the proton as predicted by the GBE (full line) and OGE (dashed line) RCQMs along the PFSM approach; in addition the results for the case with only the confinement potential (inherent in the GBE RCQM) are given (dash-dotted line). For comparison also the predictions of the II RCQM (dotted line) after ref. [11] are shown. Experimental data are from refs. [13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and [23, 24, 25, 26, 27, 28, 29].

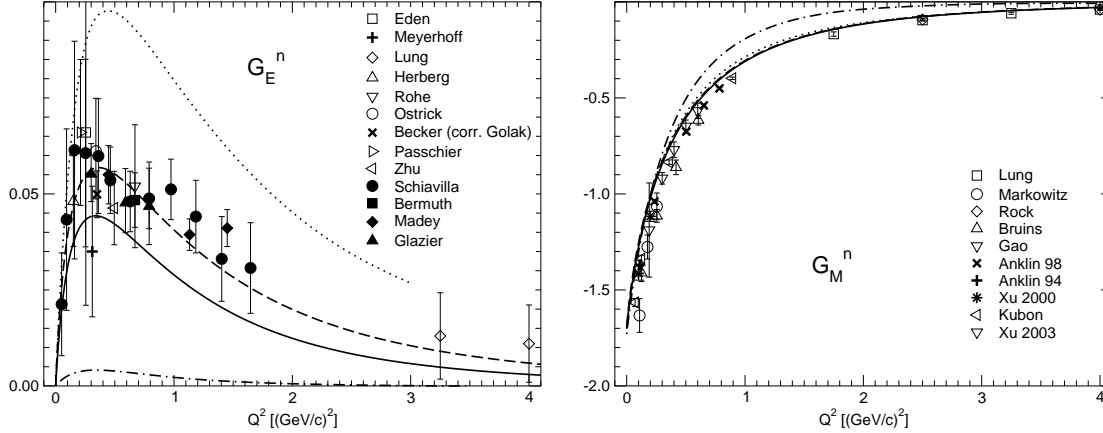


FIGURE 4. Same as in fig. 3 but for the neutron. Experimental data are from refs. [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41].

almost zero result in particular for the neutron electric form factor (see fig. 4).

A corresponding behaviour is found for the nucleon electromagnetic radii and magnetic moments: The direct PFSM predictions of the RCQMs are very close to the experimental data both for the proton and the neutron [7, 12]. Even for these observables, which relate to momentum transfers $Q^2 \rightarrow 0$, relativistic effects are high importance, and a nonrelativistic theory is by no means adequate.

Quite similar results are obtained for electric radii and magnetic moments also of the other baryon ground states, where a comparison to experiment is possible [12].

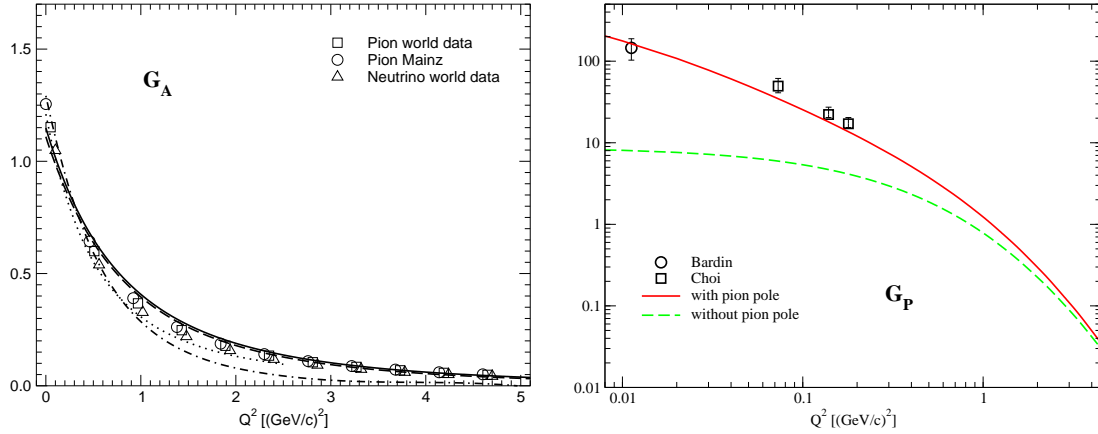


FIGURE 5. Nucleon axial and induced pseudoscalar form factors G_A and G_P , respectively. For G_A the denotation of the curves is the same as in figs. 3 and 4. The experimental data are shown assuming a dipole parameterization with the axial mass value M_A deduced from pion electroproduction (world average: squares, Mainz experiment [44]: circles) and from neutrino scattering [45] (triangles). For G_P , results from the GBE RCQM are shown with and without pion-pole contribution. The corresponding experimental data are from ref. [46].

Axial nucleon form factors

With regard to the nucleon axial and induced pseudoscalar form factors, G_A and G_P , the situation is quite similar to the elastic electromagnetic form factors [9, 42]. Only the covariant predictions of the RCQMs come close to the experimental data. Again the PFSM results and the ones obtained in the Bethe-Salpeter approach are alike [43]. The nonrelativistic impulse approximation is not acceptable. In case of the induced pseudoscalar form factor G_P the inclusion of the pion pole term is crucial. It can naturally be implemented for the GBE RCQM, in line with the dynamics in its hyperfine interaction (pseudoscalar boson exchange).

STRONG DECAYS

Recently, the relativistic study of all single-meson decay modes of the light and strange resonances below ≈ 2 GeV has been completed in the approach adopting a PFSM decay operator [47, 48, 49]. The covariant results for the partial π , η , and K decay widths show a typical behaviour in the sense that they usually underestimate the experimental data. The situation is exemplified for octet baryon resonances in fig. 6. Similar patterns are obtained for the decuplet and singlet resonances [4].

CLASSIFICATION OF BARYON STATES

The systematics found in the decay widths together with a detailed analysis of the spin-flavor contents and spatial structures of the baryon wave functions allow for a new and extended classification of the ground and resonant states into flavor multiplets [4]. The

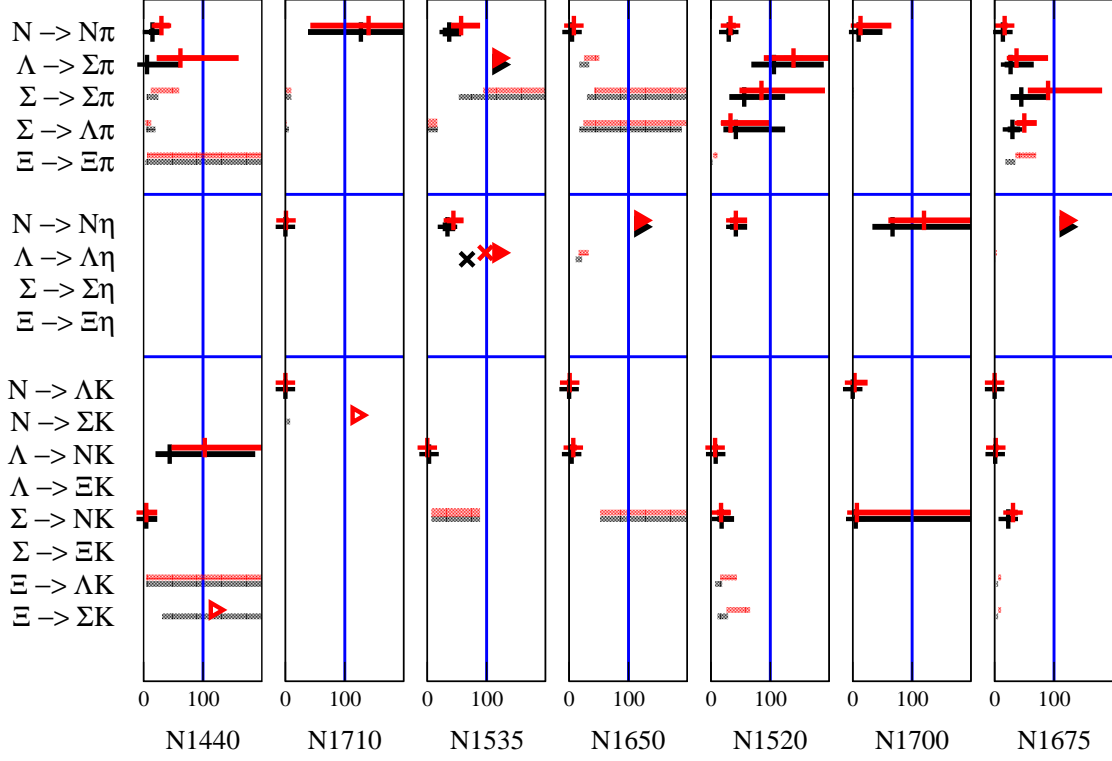


FIGURE 6. Predictions for partial π , η , and K decay widths of the GBE (black/lower entries) and OGE (red/upper entries) RCQMs from the PFSM calculation for certain flavor octets (cf. table 1 in the next section). The results shown by + crosses are presented as percentages of the best estimates for experimental data reported by the PDG [6], with the horizontal lines showing the experimental uncertainties. In case of shaded lines without crosses the PDG gives only total decay widths, and the theoretical results are represented relative to them. The triangles point to results outside the plotted range. For the particular decay $\Lambda(1670) \rightarrow \Lambda\eta$ in addition to the theoretical masses also experimental ones were used, and the corresponding results are marked by \times crosses.

members of the flavor octets, decuplets, and singlets following from our relativistic study are summarized in tables 1 to 3. In some instances we find assignments different from the ones quoted by the PDG [6]. A few resonances not considered by the PDG reasonably fit into our multiplet classification (e.g., in the Σ spectrum). Of course, additional experimental evidences would be highly welcome, and are in fact necessary, to confirm the assignments of certain states. For a thorough discussion of the detailed properties of the members in each one of the multiplets, especially also with regard to spatial probability density distributions in terms of Jacobi coordinates between the constituent quarks, see ref. [4].

SUMMARY

At present, RCQMs allow for a unified description of the light and strange baryon spectra in reasonable agreement with experiment. This is especially true for the GBE RCQM,

TABLE 1. Classification of flavor octet baryons. The denotation of the mass eigenstates is made according to the nomenclature of baryon states seen in experiment. The superscripts denote the percentages of octet content as calculated with the GBE RCQM [1]. States in bold face have either not been assigned by the PDG [6] or differ from their assignment.

$(LS)J^P$				
$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(\mathbf{1690})^{100}$
$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(\mathbf{1560})^{94}$	
$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(\mathbf{1620})^{100}$	
$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(\mathbf{1940})^{100}$	
$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(\mathbf{1950})^{100}$

TABLE 2. Classification of flavor decuplet baryons. Analogous notation as in Table 1.

$(LS)J^P$				
$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(\mathbf{1690})^{99}$		
$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(\mathbf{1750})^{94}$		
$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			

which produces the right level orderings both in the N and Λ spectra, due to its flavor-dependent hyperfine interaction. The description of the $\Lambda(1405)$, however, remains as a notorious problem.

The electroweak structure of the nucleons and other baryon ground states is well described by covariant results following from the point-form approach; they are very similar to the predictions by the II RCQM in the framework of the Bethe-Salpeter equation.

The strong decays cannot be explained by the PFSM results. Obviously considerable refinements are necessary in the decay operator and/or resonance wave functions. Still,

TABLE 3. Classification of flavor singlet baryons. Analogous notation as in Table 1.

$(LS)J^P$	
$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$
$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$
$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(\mathbf{1810})^{92}$

the consistent pattern found in the results for the partial decay widths, in combination with an analysis of the spin-flavor contents and spatial structures of the baryon wave functions, gives useful hints for the classification of resonances into $SU(3)$ flavor multiplets.

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